Chapter 4 – Trigonometric Functions

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Chapter 4: Trigonometric Functions
Topic 1: Angles and Their Measure

Vocabulary:

______________: A part of a line that has only one endpoint and extends forever in the opposite direction.

______________: Formed by two rays that have a common endpoint.

______________: The ray where the angle begins.

______________: The ray where the angle ends.

______________: The common endpoint of the two rays of an angle.

______________: When an angle's vertex is the origin and its initial side is along the x-axis.

______________: In the coordinate plane, they are generated by counterclockwise rotation.

______________: In the coordinate plane, they are generated by clockwise rotation.

______________: An angle that terminates on the x or y axis. (Common examples: 90° 180°, 270°, or 360°)

______________: Two angles with the same initial and terminal sides.

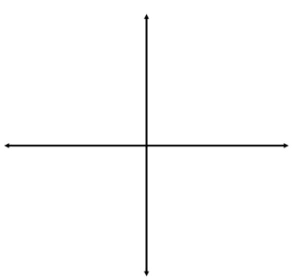
______________: Two angles whose measures sum to 90°. Only positive angles are used.

______________: Two angles whose measures sum to 180°. Only positive angles are used.

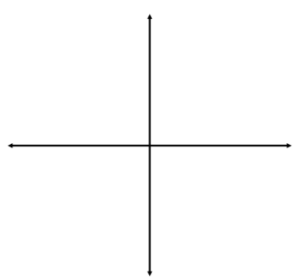
Examples using Angle Definitions

1. Sketch each angle in standard position. Label the initial and terminal sides. State the quadrant

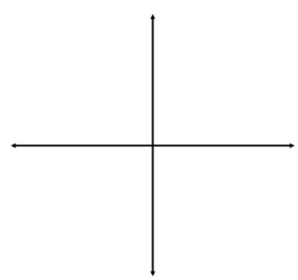
$45°$



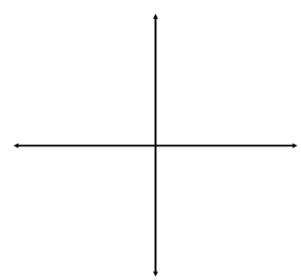
$210°$



$-135°$



$410°$



2. Find a positive coterminal angle to the following (many possible answers):

$405°$ $120°$ $-45°$ $300°$

3. Find a negative coterminal angle to the following (many possible answers):

$60°$ $-110°$ $735°$ $108°$

4. Find the standard coterminal angle (less than 360°) to the following:

$-35°$ $715°$ $-845°$ $3010°$

5. If possible, find the compliment AND supplement to the following angles:

$62°$ $123°$ $200°$ $82°$

Converting Degrees to Radians:

Assuming we are using a Unit Circle (radius of 1), the length around the outside of the circle measures exactly 2π. Using this information, we can write any angle given in degree measure as a proportion of this circle in radian measure, and vice versa. Since 360° is equivalent to 2π, we can use the identity that $180° = \pi$. Recall how to translate between the two:

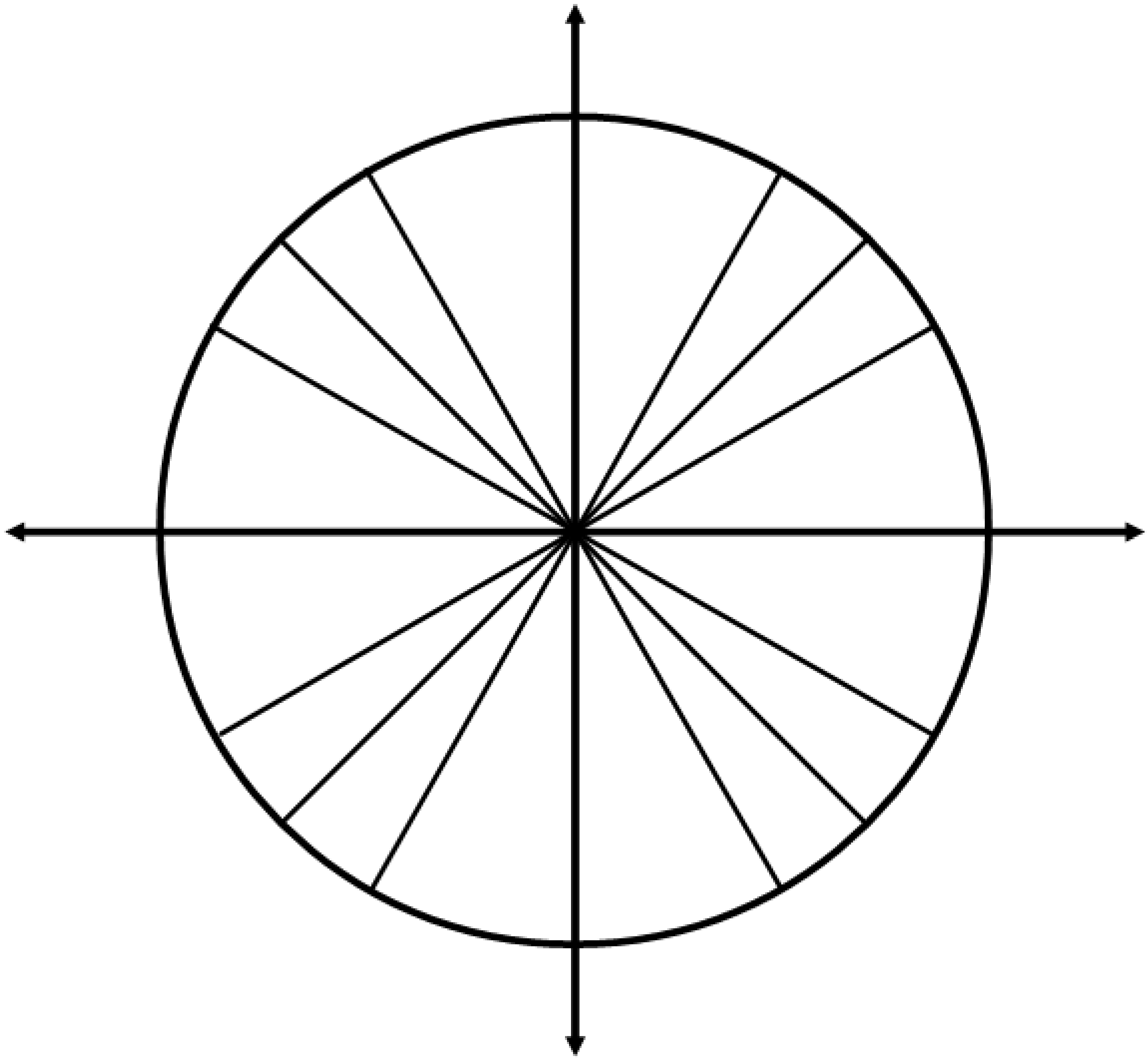
$$radians = degrees \cdot \frac{\pi}{180} \quad \text{and} \quad degrees = radians \cdot \frac{180}{\pi}$$

Examples: Fill in the missing measures

Degrees	Radians
$135°$	
$780°$	
	$\frac{3\pi}{2}$
	$-\frac{4\pi}{7}$
$-60°$	

The Unit Circle

We should be familiar with many common angles used in Trigonometry. Fill in the following unit circle, labeling all angles in both degree and radian measurement.



Arcs and Angles

Since all circles do not have a radius of 1, we must have a way to extend radian measure to larger circles. For this, we use the formula:

$$S = \theta r$$

Where: S = the length of the intercepted arc

r = the radius of the circle

θ = the measure of the central angle, in radians

Understand the formula: In the unit circle, the angle in radians exactly represents the measure of the arc around the circle because the radius being multiplied is just 1. As the circle gets larger, the angle rays travel further apart, creating a larger arc directly proportional to the distance the rays traveled (radius).

Examples:

6. A central angle θ intecepts an arc of 15 inches in a circle with a radius of 6 inches. What is the measure of the angle?

7. A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 45°. Express in terms of π, then round to two decimals places.

8. A central angle in a circle of radius 12 feet intercepts an arc of 42 feet. What is the radian measure of the angle?

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Chapter 4: Trigonometric Functions
Topic 1: Homework

In Exercises 13–20, convert each angle in degrees to radians. Express your answer as a multiple of π.

13. $45°$ **14.** $18°$ **15.** $135°$

16. $150°$ **17.** $300°$ **18.** $330°$

19. $-225°$ **20.** $-270°$

In Exercises 21–28, convert each angle in radians to degrees.

21. $\frac{\pi}{2}$ **22.** $\frac{\pi}{9}$ **23.** $\frac{2\pi}{3}$

24. $\frac{3\pi}{4}$ **25.** $\frac{7\pi}{6}$ **26.** $\frac{11\pi}{6}$

27. -3π **28.** -4π

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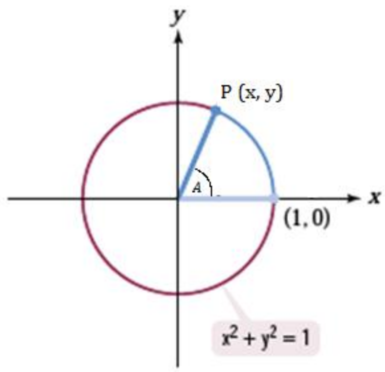
Chapter 4: Trigonometric Functions
Topic 2: The Unit Circle (Day 1)

SOH CAH TOA in the Unit Circle

With a radius of 1, the unit circle allows for easy recognition of sine, cosine, and tangent based on the coordinate point that the terminal ray of an angle makes with the unit circle.



$\sin A =$

$\cos A =$

$\tan A =$

All Six Functions

Recall three additional functions, which are the reciprocals of the three main functions

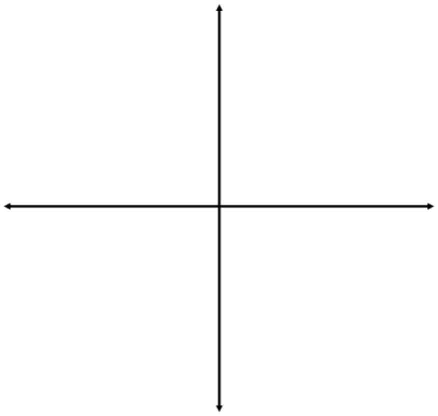
In Words:	**As a reciprocal**:	**On the unit circle**:
Cosecant: The reciprocal identity of Sine	$\csc\theta = \frac{1}{\sin\theta} = \frac{Hyp}{Opp}$	$\csc\theta =$
Secant: The reciprocal identity of Cosine	$\sec\theta = \frac{1}{\cos\theta} = \frac{Hyp}{Adj}$	$\sec\theta =$
Cotangent: The reciprocal identity of Tangent	$\cot\theta = \frac{1}{\tan\theta} = \frac{Adj}{Opp}$	$\cot\theta =$

Summarize All 6 Functions on the Unit Circle:

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$ $\cot\theta =$

<u>Example</u>: The terminal ray of a standard angle intercepts the unit circle at the point $\left(-\frac{3}{5}, -\frac{4}{5}\right)$. Sketch this angle on the unit circle (hint: which quadrant?) and identify all 6 identities.



$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$ $\cot\theta =$

How good is your memory??

$\boldsymbol{\theta}$	$\mathbf{sin}\,\boldsymbol{\theta}$	$\csc\theta$	$\mathbf{cos}\,\boldsymbol{\theta}$	$\sec\theta$	$\mathbf{tan}\,\boldsymbol{\theta}$	$\cot\theta$
30°						
45°						
60°						

$\boldsymbol{\theta}$	$\mathbf{sin}\,\boldsymbol{\theta}$	$\csc\theta$	$\mathbf{cos}\,\boldsymbol{\theta}$	$\sec\theta$	$\mathbf{tan}\,\boldsymbol{\theta}$	$\cot\theta$
0°, 360°						
90°						
180°						
270°						

Your Homework tonight is to memorize this, it will help greatly.

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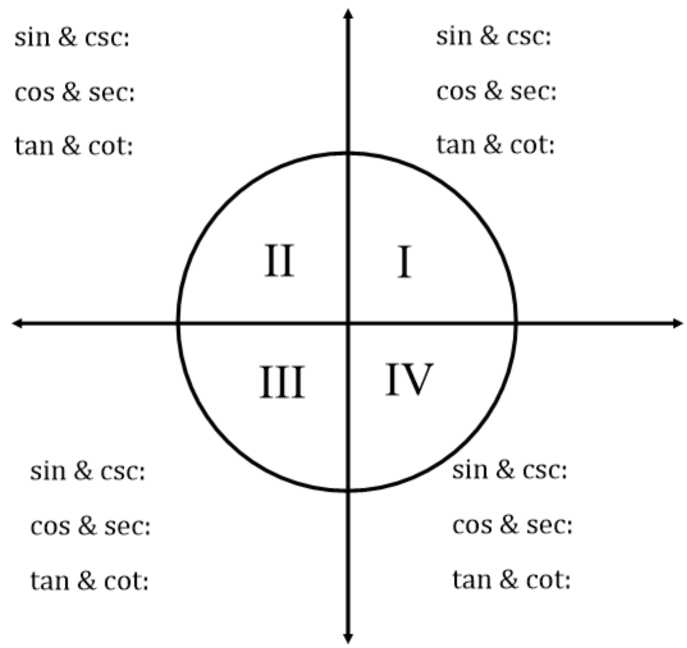
Chapter 4: Trigonometric Functions
Topic 2: The Unit Circle (Day 2)

Recall: Which functions are positive in which quadrants? Let the fact that cosine relates to x and sine relates to y drive your answers.

Where is x positive or negative? Then so is cosine.

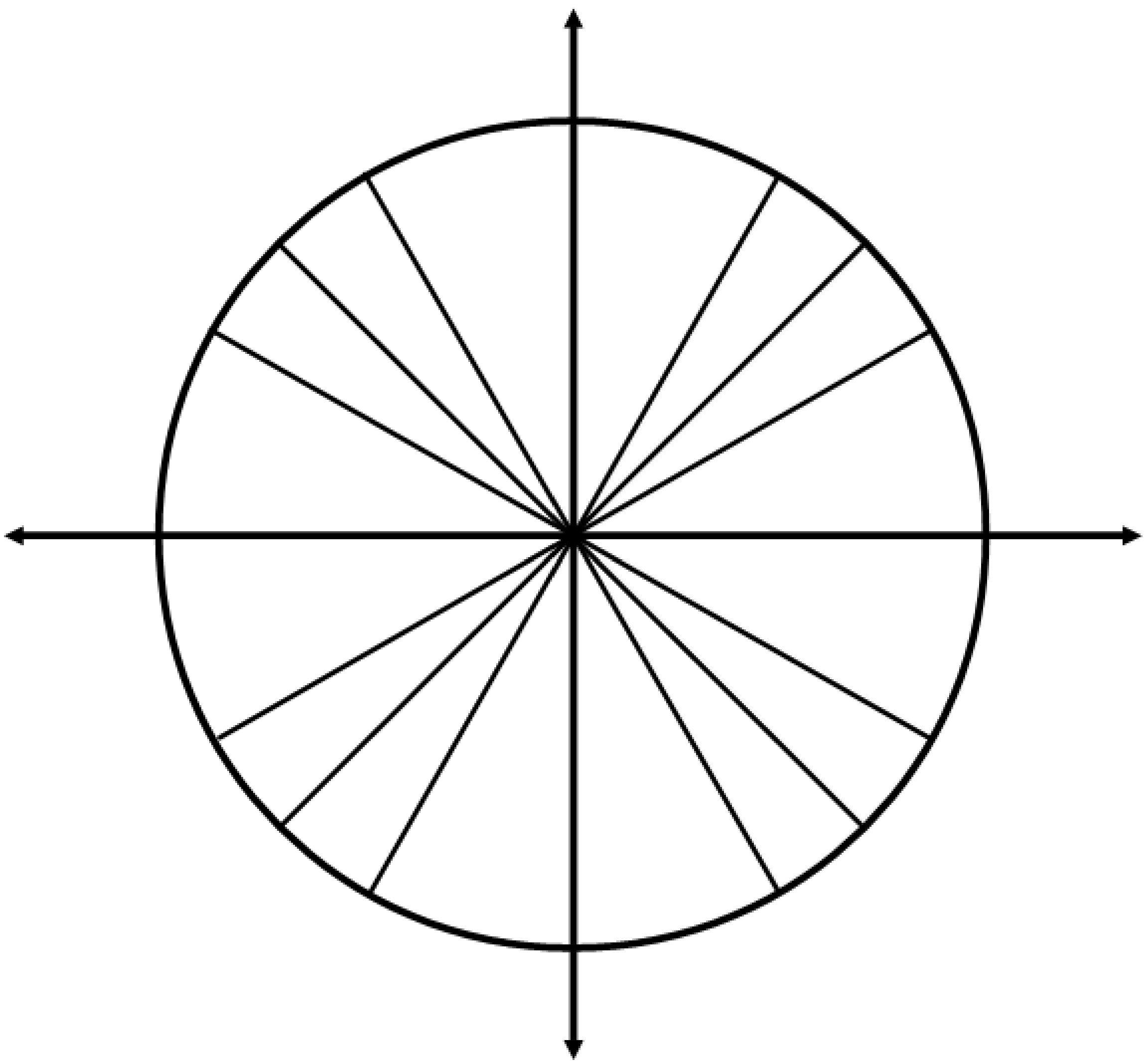
Where is y positive or negative? Then so is sine.

Recall that Tangent divides the two.



Full Unit Circle

- Write each angle in degree and radian form
- Identify the coordinate point where each angle intercepts the circle



Pythagorean Identities

The equation of the unit circle is represented as $x^2 + y^2 = 1$

Knowing what we do about sine and cosine, this can be written as the Pythagorean Identity:

$$\boldsymbol{\cos^2\theta + \sin^2\theta = 1}$$

With further manipulation, we can write this exact same identity in two other ways:

- Staring from the main pythagorean identity, divide each term by $\cos^2\theta$.

- Starting from the main pytahtroean identity, divide each term by $\sin^2\theta$.

These identities help us to find the missing trig ratios for an angle. Pick which of the pythagorean identities to use based on what you already know.

Examples:

Given that θ terminates in Quadrant I, and $\sin\theta = \frac{3}{5}$, find $\cos\theta$ using a pythagorean identity.

Given that θ terminates in Quadrant II and $\cos\theta = \frac{-5}{13}$, find $tan\theta$ using a pythagorean identitiy.

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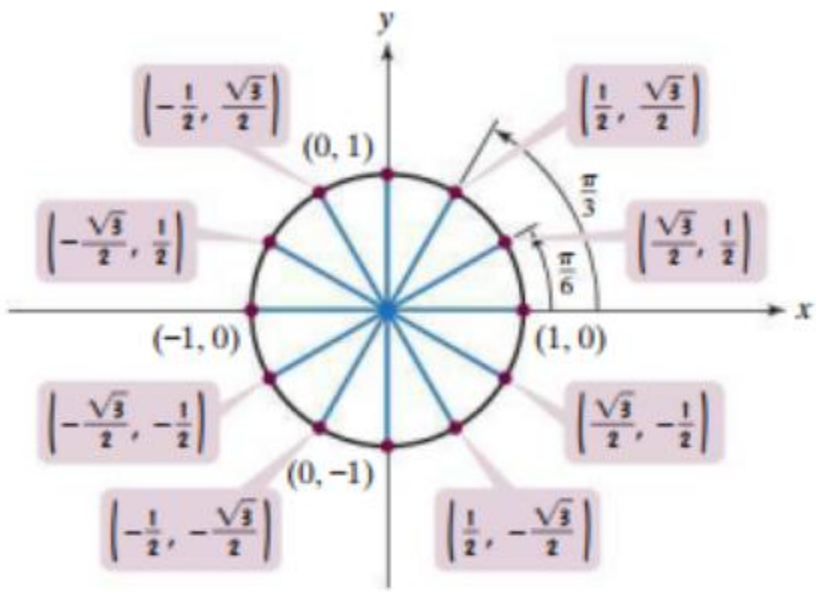
Chapter 4: Trigonometric Functions
Topic 2: Homework

In Exercises 5–18, the unit circle that follows has been divided into twelve equal arcs, corresponding to t-values of

$$0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{11\pi}{6}, \text{ and } 2\pi.$$

Use the (x, y) coordinates in the figure to find the value of each trigonometric function at the indicated real number, t, or state that the expression is undefined.



5. $\sin \frac{\pi}{6}$ **6.** $\sin \frac{\pi}{3}$ **7.** $\cos \frac{5\pi}{6}$

8. $\cos \frac{2\pi}{3}$ **9.** $\tan \pi$ **10.** $\tan 0$

11. $\csc \frac{7\pi}{6}$ **12.** $\csc \frac{4\pi}{3}$ **13.** $\sec \frac{11\pi}{6}$

14. $\sec \frac{5\pi}{3}$ **15.** $\sin \frac{3\pi}{2}$ **16.** $\cos \frac{3\pi}{2}$

17. $\sec \frac{3\pi}{2}$ **18.** $\tan \frac{3\pi}{2}$

In Exercises 25–28, sin t *and* cos t *are given. Use identities to find* tan t, csc t, sec t, *and* cot t. *Where necessary, rationalize denominators.*

25. $\sin t = \frac{8}{17}, \cos t = \frac{15}{17}$ **26.** $\sin t = \frac{3}{5}, \cos t = \frac{4}{5}$

27. $\sin t = \frac{1}{3}, \cos t = \frac{2\sqrt{2}}{3}$ **28.** $\sin t = \frac{2}{3}, \cos t = \frac{\sqrt{5}}{3}$

In Exercises 29–32, $0 \leq t < \frac{\pi}{2}$ *and* sin t *is given. Use the Pythagorean identity* $\sin^2 t + \cos^2 t = 1$ *to find* cos t.

29. $\sin t = \frac{6}{7}$ **30.** $\sin t = \frac{7}{8}$

31. $\sin t = \frac{\sqrt{39}}{8}$ **32.** $\sin t = \frac{\sqrt{21}}{5}$

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Chapter 4: Trigonometric Functions
Topic 3: Right Triangle Trig

Cofunctions

Another relationship among the 6 Trig Functions is based on the complements of the angle involved. These functions are paired up as Cofunctions.

Examples of Cofunctions:

sine - cosine

tangent - cotangent

secant - cosecant

Notice that the pairing is different than inverses!

An important relationship between these pairs is that:

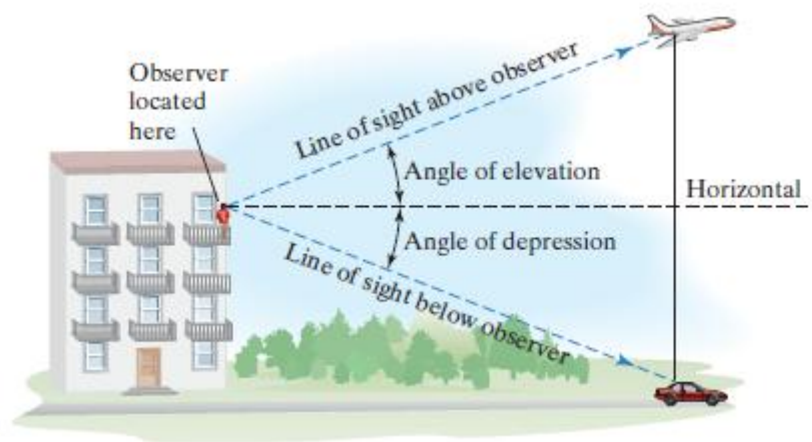
Recall! Complements: Two angles which add up to _________ or ________ radians.

Examples: Using cofunctions, find an equivalent value for each question below.

1. $\sin 72° = \cos$ ____
2. 2. $\csc\frac{\pi}{3}$
3. $\cot\frac{\pi}{12}$
4. 4. $\sin 46°$

Applied Right Angle Trig

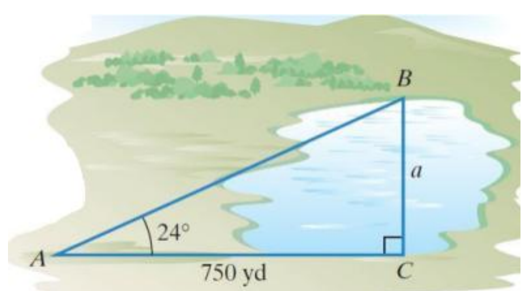
- Sketch a picture, utilizing a right triangle
- Label as much as possible, including all given information
- Choose your function & setup
- Solve



Examples:

1. Sighting the top of a building, a surveyor measured the angle of elevation t be 22°. His tripod is 5 feet above the ground and 300 feet from the building. Find the building's height.

2. The distance across a lake is unknown. To find the distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?



3. A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the sun to the nearest degree.

4. A flagpole that is 14 meters tall casts a shadow 10 meters long. Find the angle of elevation of the sun to the nearest degree.

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Chapter 4: Trigonometric Functions
Topic 3: Homework

In Exercises 21–28, find a cofunction with the same value as the given expression.

21. $\sin 7°$

22. $\sin 19°$

23. $\csc 25°$

24. $\csc 35°$

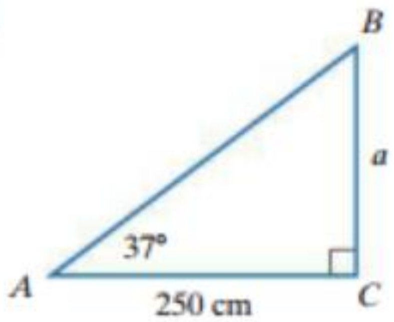
25. $\tan \frac{\pi}{9}$

26. $\tan \frac{\pi}{7}$

27. $\cos \frac{2\pi}{5}$

28. $\cos \frac{3\pi}{8}$

29.



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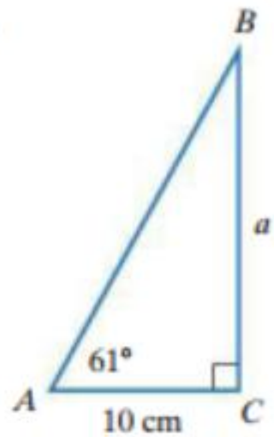
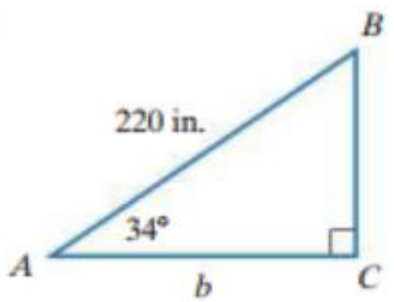


31.



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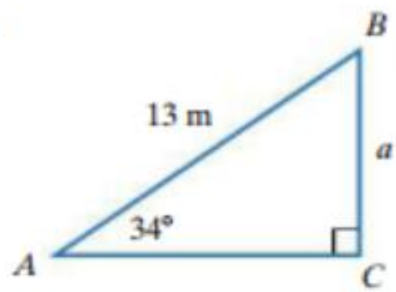

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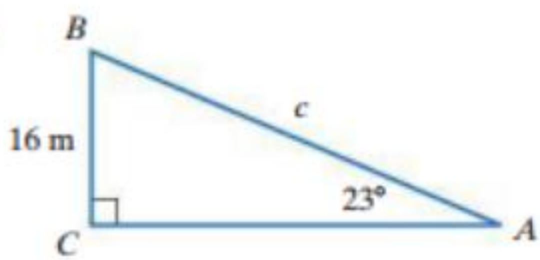

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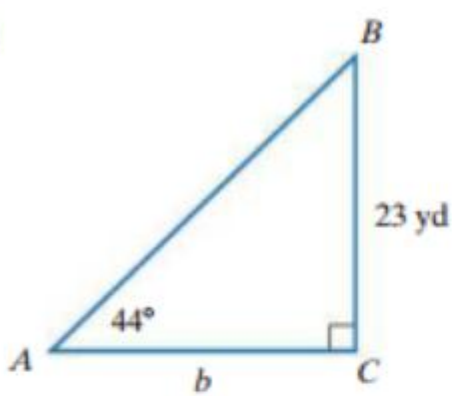

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Chapter 4: Trigonometric Functions
Topic 4: Trig Functions of Any Angle

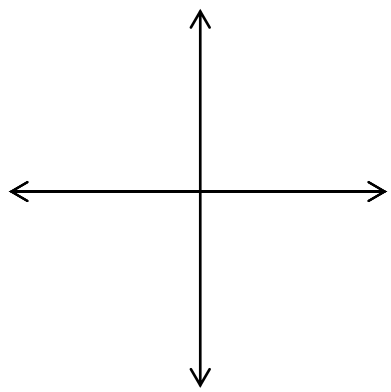
Do Now:

Identify the quadrant in which the following angles must terminate.

1. $\sin\theta > 0, \cos\theta < 0$
2. $\tan\theta > 0, \csc\theta > 0$
3. $\csc\theta < 0, \cot\theta < 0$
4. $\sec\theta < 0, \tan\theta > 0$

Definition: Reference Angle

Let θ be a non-acute angle in standard position that lies in standard form a quadrant. Its reference angle is the positive acute angle formed by the terminal side of θ and the $\boldsymbol{x-axis}$.



Examples: Find the reference angle for each of the following angles.

Hint – sometimes you'll need to think of a coterminal angle first!

5. $\theta = 345°$
6. $\theta = \frac{5\pi}{6}$
7. $\theta = -135°$
8. $\theta = \frac{7\pi}{4}$
9. $\theta = 455°$
10. $\theta = \frac{11\pi}{3}$

Using Reference Angles to find Exact Values

By pulling together all information about a non-acute angle can help us determine exact values of their trig functions. When asked for the exact value of an angle larger than 90˚, gather information about:

Q *Which quadrant does the angle belong to?*

S F *Is the function positive or negative here?*

R *What is the reference angle?*

Examples: Find the exact value of the following.

11. $\sin 135°$

12. $\cos \frac{4\pi}{3}$

13. $\cot\left(-\frac{\pi}{3}\right)$

14. $\sin 300°$

15. $\tan \frac{5\pi}{4}$

16. $\sec \frac{11\pi}{6}$

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Chapter 4: Trigonometric Functions
Topic 4: Homework

In Exercises 1–8, a point on the terminal side of angle θ is given. Find the exact value of each of the six trigonometric functions of θ.

1. $(-4, 3)$ **2.** $(-12, 5)$ **3.** $(2, 3)$
4. $(3, 7)$ **5.** $(3, -3)$ **6.** $(5, -5)$
7. $(-2, -5)$ **8.** $(-1, -3)$

In Exercises 23–34, find the exact value of each of the remaining trigonometric functions of θ.

23. $\cos\theta = -\frac{3}{5}$, θ in quadrant III
24. $\sin\theta = -\frac{12}{13}$, θ in quadrant III
25. $\sin\theta = \frac{5}{13}$, θ in quadrant II
26. $\cos\theta = \frac{4}{5}$, θ in quadrant IV
27. $\cos\theta = \frac{8}{17}$, $270° < \theta < 360°$
28. $\cos\theta = \frac{1}{3}$, $270° < \theta < 360°$
29. $\tan\theta = -\frac{2}{3}$, $\sin\theta > 0$ **30.** $\tan\theta = -\frac{1}{3}$, $\sin\theta > 0$

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Chapter 4: Trigonometric Functions
Topic 5: Inverse Functions

Inverse Functions

The inverse of sine is notated as ____________. I do this on my calculator by ____________.

The inverse of cosine is notated as ____________. I do this on my calculator by ____________.

The inverse of tangent is notated as ____________. I do this on my calculator by ____________.

The inverse functions answer the question "what angle has this value?"

The inverse function is NOT the same as the reciprocal functions!

Examples: Assume Quadrant I angles.

1. Find the value of the angle to the nearest hundredth. Assume a Quadrant I angle.

   $cos^{-1}(0.87)$ $tan^{-1}(1.49)$

2. Find the value to the nearest hundredth. Assume a Quadrant I angle.

   $cos\left(tan^{-1}(8.6)\right)$ $sin\left(cos^{-1}(0.11)\right)$

3. Without using a calculator, evaluate the exact value of each.

   $sin\left(sin^{-1}(0.52)\right)$ $tan\left(tan^{-1}(11.6)\right)$

   $sin^{-1}\left(sin\left(\frac{3\pi}{2}\right)\right)$ $cos^{-1}\left(cos\left(\frac{\pi}{6}\right)\right)$

Composition of Trigonometric Functions

Common Angles:

To evaluate a composition of trigonometric functions, we follow a similar protocol to when we evaluate the composition of any other function. We work inside to out, where we treat the answer to the inside, as the input to the outside.

Without using a calculator, evaluate the exact value of each. (Commonly used angles)

$\sin\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)$

$\cos(\sin^{-1}(-1))$

$\tan\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

$\cos(\sin^{-1}(0.5))$

Uncommon Angles:

When we are looking at uncommon angles, we still work inside to out, however we will be using SOH CAH TOA to find the answer. We use the values of the inside trig function to make a triangle, then we choose the appropriate values based on the outside function.

Without using a calculator, evaluate the exact value of each. (non-commonly used angles)

$\tan\left(\cos^{-1}\left(\frac{8}{17}\right)\right)$

$\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$

$\sin\left(\tan^{-1}\left(\frac{7}{24}\right)\right)$

$\csc\left(\tan^{-1}\left(\frac{40}{9}\right)\right)$

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Chapter 4: Trigonometric Functions
Topic 5: Homework

In Exercises 1–18, find the exact value of each expression.

1. $\sin^{-1}\frac{1}{2}$

2. $\sin^{-1}0$

3. $\sin^{-1}\frac{\sqrt{2}}{2}$

4. $\sin^{-1}\frac{\sqrt{3}}{2}$

5. $\sin^{-1}\left(-\frac{1}{2}\right)$

6. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

7. $\cos^{-1}\frac{\sqrt{3}}{2}$

8. $\cos^{-1}\frac{\sqrt{2}}{2}$

9. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

10. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

11. $\cos^{-1}0$

12. $\cos^{-1}1$

13. $\tan^{-1}\frac{\sqrt{3}}{3}$

14. $\tan^{-1}1$

15. $\tan^{-1}0$

16. $\tan^{-1}(-1)$

17. $\tan^{-1}(-\sqrt{3})$

18. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

In Exercises 31–46, find the exact value of each expression, if possible. Do not use a calculator.

31. $\sin(\sin^{-1} 0.9)$

32. $\cos(\cos^{-1} 0.57)$

33. $\sin^{-1}\left(\sin \frac{\pi}{3}\right)$

34. $\cos^{-1}\left(\cos \frac{2\pi}{3}\right)$

35. $\sin^{-1}\left(\sin \frac{5\pi}{6}\right)$

36. $\cos^{-1}\left(\cos \frac{4\pi}{3}\right)$

37. $\tan(\tan^{-1} 125)$

38. $\tan(\tan^{-1} 380)$

39. $\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]$

40. $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right]$

41. $\tan^{-1}\left(\tan \frac{2\pi}{3}\right)$

42. $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$

43. $\sin^{-1}(\sin \pi)$

44. $\cos^{-1}(\cos 2\pi)$

45. $\sin(\sin^{-1} \pi)$

46. $\cos(\cos^{-1} 3\pi)$

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Chapter 4: Trigonometric Functions
Topic 6: Applications of Trig Functions

Using Two Right Triangles to Solve a Problem

Keeping in mind the basic SOHCAHTOA relationships and that drawing accurate pictures can help.

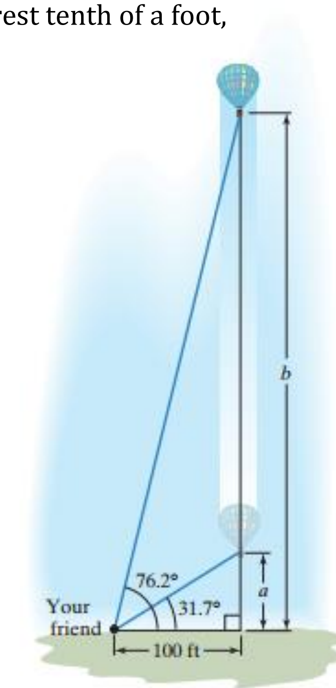
First Some Basic Examples:

From a point on level ground 125 feet from the base of a tower, the angle of elevation is 57.2°. Approximate the height of the tower to the nearest foot.

A kite flies at a height of 30 feet when 65 feet of string is out. If the string is in a straight line, find the angle that it makes with the ground. Round to the nearest tenth of a degree.

Now Let's Combine Them:

Example #1: You are taking your first hot-air balloon ride. Your friend is standing on level ground, 100 feet away from your point of launch. At one instant, the angle of elevation from the video camera to your face is 31.7°. One minute later, the angle of elevation is 76.2°. How far did you travel, to the nearest tenth of a foot, during that minute?



Solve for a:

Solve for b:

How far did you travel?

Example #2: You are standing on level ground 800 feet from Mt. Rushmore, looking at the sculpture of Abraham Lincoln's face. The angle of elevation to the bottom of the sculpture is 32° and the angle of elevation to the top is 35°. Find the height of the sculpture of Lincoln's face to the nearest tenth of a foot.

Name: ____________________ Date: ____________ Period: ______

Chapter 4: Trigonometric Functions
Topic 6: Homework

41. The tallest television transmitting tower in the world is in North Dakota. From a point on level ground 5280 feet (1 mile) from the base of the tower, the angle of elevation is 21.3°. Approximate the height of the tower to the nearest foot.

43. The Statue of Liberty is approximately 305 feet tall. If the angle of elevation from a ship to the top of the statue is 23.7°, how far, to the nearest foot, is the ship from the statue's base?

45. A helicopter hovers 1000 feet above a small island. The figure shows that the angle of depression from the helicopter to point P on the coast is 36°. How far off the coast, to the nearest foot, is the island?